Abstraction Damage Estimation of Sediment Bypass Tunnels: 
Validation and Comparison of two Prediction Models

Christian AUEL, Robert M. BOES(1) and Tetsuya SUMI
(1) Laboratory of Hydraulics, Hydrology and Glaciology, ETH Zurich, Switzerland

Synopsis
Two abrasion prediction models developed for hydraulic structures prone to supercritical flows are compared herein: The models by Ishibashi (1983) and Auel et al. (2015). Both models are compared and validated using prototype data from the Asahi sediment bypass tunnel in Japan. The prediction model by Ishibashi (1983) contains some inconsistencies in its grinding stress term being much higher than its particle impact term which contradicts statements by other researchers. However, it is shown herein that the model leads to good results if only the particle impact term is considered. The prediction model by Auel et al. (2015) shows excellent performance if the abrasion coefficient $k_v$ is adjusted to prototype measurements. The analysis presented herein leads to the conclusion that $k_v$ for concrete is lower than $10^6$ being a widely accepted value for bedrock abrasion. Independent of the prediction model, utmost interest has to be laid on the estimation of the sediment transport. The theoretical transport capacity may be much higher than the real transport and should be calibrated using prototype field data such as reservoir bed topography surveys.

Keywords: Reservoir sedimentation, Sediment bypass tunnel, Abrasion prediction model, Hydro-abrasive wear

1 Introduction
The use of sediment bypass tunnels (SBTs) is an effective strategy to counteract reservoir sedimentation. During flood events sediment-laden flows are bypassed around the dam into the tailwater inhibiting sedimentation in the reservoir. Due to high flow velocities in combination with bedload sediment transport, many SBTs worldwide are prone to hydro-abrasive wear as shown in Fig. 1 (Jacobs et al. 2001, Sumi et al. 2004, Auel and Boes 2011, Boes et al. 2014, Nakajima et al. 2015, Baumer and Radogna 2015, Müller and Walker 2015).
Abstraction is a wear phenomenon involving progressive material loss due to hard particles forced against and moving along a solid surface. In bedrock rivers, abrasion is the driving process for bed incision (Sklar and Dietrich 2004, 2006, Lamb et al. 2008, Turowski 2009) while at hydraulic structures such as spillways, weirs, flushing channels and SBTs abrasion causes severe damage of the invert surface. In general, abrasive damage can always be expected when particle bedload transport takes place. Particles are transported in sliding, rolling or saltation motion depending on the flow conditions causing grinding, rolling or saltating impact stress on the bed. The vertical component of a saltating particle causes the so called deformation wear, which is related to the particle impact, whereas the horizontal component causes cutting wear, which is related to grinding stress (Bitter 1963a, 1963b). Engel (1978) stated that erosion depends on the sine of the impact angle because the magnitude of the peak tensile stress varies with the normal component of the impact velocity, i.e. the vertical velocity component is the driving factor. Cutting wear caused by the horizontal velocity component is important in ductile materials and in case...
of highly angular impacting particles, but is not significant when brittle materials are impacted by rounded grains as present in river systems (Head and Harr 1970, Sklar and Dietrich 2004). A number of models exist to predict abrasion. While the models for the prediction of bedrock incision rate (Sklar and Dietrich 2004, Lamb et al. 2008) focus on typical flow conditions in river systems in the sub- and low supercritical flow regime, the others for prediction of abrasion on hydraulic structure surfaces (Ishibashi 1983, Helbig and Horlacher 2007, Auel et al. 2015) have to account for highly supercritical flows. In this paper, the abrasion prediction model of Ishibashi (1983) is explained in detail and compared to a model recently developed by Auel et al. (2015).

Fig. 1 Concrete invert abrasion at Asahi SBT, Japan (photo: C. Auel).

2 Abrasion prediction models

2.1 Ishibashi model

A widely applied formula in Japan to predict abrasion is given by Ishibashi (1983). He considered both deformation and cutting wear due to the fact that ductile materials such as steel were also investigated besides the brittle material concrete. As the original work is written in Japanese, a short description of the developed equations is given hereafter. Ishibashi (1983) calculated the abraded invert volume $V_a$ as:

$$V_a = C_1 E_k + C_2 W_f \quad [m^3] \quad [1]$$

where $E_k$ = total particle kinetic energy by saltating particles, $W_f$ = total friction work by grinding particles, and $C_1$ and $C_2$ = material property constants [$m^2/(kgf)$] given in Table 1. Note that the unit kilogram force corresponds to Newton as 1 kgf = 9.806 N. The total kinetic energy $E_k$ is given by:

$$E_k = 1.5 V_o \sum E_i n_i \quad [kgf m] \quad [2]$$

and the total friction work $W_f$ by:

$$W_f = 5.513 \mu s V_o \sum V_{ts} E_i N_i n_i \quad [kgf m] \quad [3]$$

where $V_o$ = amount of transported sediment [$m^3$], $\mu_s$ = 0.3 = dynamic friction coefficient, $E_i$ = kinetic energy of a single particle, $\gamma_{im}$ = particle impact angle, $N_i = L/L_p$ = impact frequency, with $L_p$ = particle saltation length and $L$ = total invert length, and $n_i$ = amount of particles per volume. The value $n_i$ is calculated as:

$$n_i = \frac{(1-\lambda_p) \rho_s}{M_p} \quad [1/m^2] \quad [4]$$

where $\lambda_p$ = 0.4 = air porosity, $\rho_s$ = particle density and $M_p = 1/6\pi \rho_s D^3$ = particle mass, with $D$ = mean particle diameter. The single impact energy $E_i$ is calculated as:

$$E_i = \beta F_i^{5/3} \quad [kgf m] \quad [5]$$

where $F_i$ = impact force, and $\beta$ = auxiliary parameter given as:

$$\beta = \left[2.5 n_i^{2/3} \left(D/2\right)^{3/3} \right]^{-1} \quad [m/(kgf)^{2/3}] \quad [6]$$

where $n_i$ = auxiliary parameter given as:

$$n_i = \frac{4}{3(k_1 + k_2)} \quad [kgf/m^2] \quad [7]$$

where $k_1$ and $k_2$ = auxiliary parameter accounting for the Young’s modulus and Poisson’s ratio of both the particle and invert materials, respectively. Ishibashi (1983) proposed a constant value of $n_i = 2.41 \times 10^9$ kgf/m² for sediment gravel transported over concrete. The abrasion prediction model is based on laboratory data published in Ishibashi and Isobe (1968). Open-channel flow experiments were conducted in a 9 m...
long, 0.2 m wide and 0.2 m high laboratory flume to investigate the impact forces and saltation trajectories of bedload particles at supercritical flow conditions. Correlations for the particle saltation length \(L_p\), impact force \(F_i\), and impact angle \(\gamma_{im}\) were proposed by Ishibashi (1983) based on Ishibashi and Isobe (1968). \(L_p\) follows:

\[
L_p = 100(\theta - \theta_c)^{1.21}
\]

with \(\theta = U^*/[(s-1)gD] = \text{Shields parameter}, U_* = \text{friction velocity}, s = \rho_s/\rho = \text{fluid density}, \) and \(\theta_c = \text{critical Shields parameter}. \theta_c\) for fixed beds with low relative roughness heights is given by Ishibashi (1983) based on Novak and Nalluri (1975) as:

\[
\theta_c = \frac{2}{5} \left( \frac{s-1}{gD^3} \right)^{0.5}
\]

where \(D_* = \left[(s-1)gD\right]^{0.5}/v = \text{dimensionless grain size with } v = \text{kinematic viscosity}. \) The impact force \(F_i\) is given as follows:

\[
F_i = 1.95 \times 10^3 M_p^* (\theta - \theta_c) [\text{kgf}]
\]

where \(M_p^* = 1/6\pi(\rho_s-1000)D^3 = \text{submerged particle weight in [kg].}\) The impact angle is approximated as:

\[
\gamma_{im} = 19.2 (\theta - \theta_c)^{-0.13} [-]
\]

where \(V_s\) = vertical particle settling velocity in still water, and \(U_p\) = horizontal particle velocity.

### Tab. 1 Material property constants (Ishibashi 1983)

<table>
<thead>
<tr>
<th>Material</th>
<th>(C_1 [\text{m}^2/(\text{kgf})])</th>
<th>(C_2 [\text{m}^2/(\text{kgf})])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>(1.189 \times 10^{-7})</td>
<td>(1.135 \times 10^{-8})</td>
</tr>
<tr>
<td>Steel (SM 41)</td>
<td>(3.73 \times 10^{-11})</td>
<td>(6.59 \times 10^{-11})</td>
</tr>
<tr>
<td>Steel (HT 80)</td>
<td>(2.53 \times 10^{-11})</td>
<td>(4.78 \times 10^{-11})</td>
</tr>
<tr>
<td>Steel (SUS 304)</td>
<td>(2.04 \times 10^{-11})</td>
<td>(3.25 \times 10^{-11})</td>
</tr>
<tr>
<td>Steel (SCMnH 11)</td>
<td>(1.18 \times 10^{-10})</td>
<td>(1.33 \times 10^{-11})</td>
</tr>
</tbody>
</table>

### 2.2 Auel model

Based on open-channel flow experiments in a laboratory flume conducted by Auel (2014), an abrasion prediction model applicable for hydraulic structures prone to supercritical flows was proposed by Auel et al. (2015). The model is based on the state-of-the-art saltation abrasion model by Sklar and Dietrich (2004) developed to estimate abrasion in bedrock river systems. The abrasion rate \(A_r\) is calculated as:

\[
A_r = \frac{Y_m W_{im}^2 I \cdot q_s}{k_r f_i^2} [\text{m/s}]
\]

where \(k_r = \text{abrasion resistance coefficient}, Y_m = \text{Young’s Modulus of elasticity of the invert material [Pa]}, f_i = \text{invert material splitting tensile strength [Pa]}, W_{im} = \text{mean vertical particle impact velocity [m/s]}, I = \text{impact rate [1/m]}, \) and \(q_s = \text{specific gravimetric bedload rate [kg/(sm)]}. \) The number of impacts per unit length \(I\) is defined as the reciprocal value of the particle saltation length \(L_p\) as:

\[
I = \frac{1}{L_p} \left[\frac{1 - (U_i/U_s)^2}{(1 - P_R)^3}\right]^{0.5} (1 - P_R) [1/\text{m}]
\]

where \(P_R\) = rolling probability. The numerator of the first term on the right hand side is proposed by Sklar and Dietrich (2004) and accounts for the mode shift from saltation to suspension. The rolling probability \(P_R\) follows (Auel 2014):

\[
P_R = 8.5 \times 10^{-4} \left(\frac{\theta (k_D)}{D}\right)^{0.55}
\]

with \(k = \text{geometric roughness height and } D = \text{mean particle diameter}. \) The hop length \(L_p\) for supercritical flows is given with (Auel 2014, Auel et al. 2015):

\[
L_p/D = 251 \theta
\]

Auel et al. (2015) found a simple linear correlation of \(W_{im}\) to the friction velocity \(U_*\) as

\[
W_{im} = U_* [\text{m/s}]
\]

Additionally, Auel et al. (2015) analyzed bedrock abrasion laboratory experiments and compared them to concrete abrasion tests. It was suggested to use the abrasion coefficient value \(k_r = 10^6\) given by Sklar and Dietrich (2004) for bedrock abrasion as well as for concrete structures. Based on field data of Asahi SBT, it is shown in the following that \(k_r\) varies from the above mentioned value and should be adjusted for concrete inverts accordingly.

### 3 Procedure of model comparison

Comparison of the two prediction models is done by analyzing prototype data of the Asahi River, reservoir and SBT in Kii Peninsula, Japan. The reservoir is operated by Kansai Electric Power Co. Inc. and is part of
the Okuyoshino pump storage hydropower scheme together with the upper Seto dam. The reservoir has been facing severe sedimentation since its inauguration in 1978 due to large flood events caused by heavy rainfall and typhoons (Harada et al. 1997, Nakajima et al. 2015). To counter the ongoing sedimentation, a SBT was constructed in 1998 with a design discharge of \( Q_d = 140 \) m\(^3\)/s. The total tunnel length of 2,383.5 m includes an 18.5 m long steel-lined inlet section, a 2,350 m long concrete-lined tunnel and a 15 m long concrete-lined outlet. The tunnel is \( b = 3.80 \) m wide with a slope of \( S_b = 0.029 \). Since its inauguration the tunnel faces severe abrasion up to several decimeters. Kansai Electric conducts regular invert abrasion measurements on an annual basis (Fig. 2).

![Fig. 2 Cumulative abrasion pattern of Asahi SBT from 1998 to 2011. Flow from right to left (adapted from Nakajima et al. 2015).](image)

Abrasion is measured manually every 2 m in the streamwise and every 10 cm in the lateral direction. In case of deep scour holes in between the 2 m sections, the streamwise distance is adapted accordingly. The tunnel invert was originally lined with a concrete of compression strength \( f_c = 30 \) MPa. During annual maintenance works the invert was successively lined with 70 MPa concrete. The annual ratio of old to new concrete is given in Fig. 3 (Nakajima et al. 2015).

![Fig. 3 Percentage of high performance concrete \((f_c = 70 \) MPa) on tunnel invert (adopted from Nakajima et al. 2015).](image)

The discharge in the upstream Asahi River reach is given on an hourly basis. The catchment is rather steep with a river bed slope varying from \( S_b = 0.03 \) to 0.10 and river widths varying from about 20 to 40 m. The transported sediment has been calculated by Newjec Inc. by means of a 1D numerical model using the bedload formula of Ashida and Michiue (1972) and calibrated with data of annual reservoir bed topography surveys (Harada et al. 1997, Kataoka 2000, 2003). As a result Newjec provided a correlation of the river discharge to the amount of transported sediment which is used in the presented calculations.

![Fig. 4 Abraded invert volume and bypassed sediment volume of Asahi SBT from 1998 to 2011.](image)

4 Results and discussion

Calculations for both models are made using a mean grain size diameter of \( D = 7.5 \) cm to estimate both the sediment transport and the abrasion volume. A fraction-wise calculation considering the grain size distribution was not carried out.

4.1 Ishibashi model

The results of the abrasion prediction model given in
Eq. (1) by Ishibashi (1983) are shown in Fig. 5. The kinetic impact and grinding stress terms $C_1E_k$ and $C_2W_f$, respectively, are separated to better understand their effects. The total abraded volume $V_a$ is the sum of the two terms. To compare the results the relative change $\delta$ is calculated as

$$\delta = \frac{V_{a, \text{estimate}} - V_{a, \text{real}}}{V_{a, \text{real}}} \cdot 100 \ [%] \quad [17]$$

The results show that the abrasion volume is largely overestimated by $\delta = 190 \pm 118\%$ considering both terms. The grinding stress term is in average 2.6 times higher than the kinetic impact term. Considering the fact that $W_f$ has a negligible effect in case of brittle materials (Head and Harr 1970) and is neglected by other researchers (Sklar and Dietrich 2001, 2004, Auel et al. 2015), the value should be at least smaller than $C_1E_k$. Additionally, Auel (2014) showed for an exemplary calculation that the energy induced by rolling and sliding motion of a particle on a concrete surface is about 10% and 0.1%, respectively, compared to the kinetic energy by saltating particles. Hence, the conclusion may be drawn that the grinding stress term in Eq. (1) is only appropriate in case of ductile materials and should not be considered in case of brittle inverts such as concrete. It is proposed to omit the grinding stress term and rewrite Eq. (1) as:

$$V_a = C_1 \cdot 1.5V_{a} \sum E_iN_i n_i \quad [\text{m}^3] \quad [18]$$

Fig. 5 reveals a good agreement of the measured to the estimated data applying only the kinetic energy as given in Eq. (18). The average deviation over 14 years is only $\delta = 35 \pm 23\%$.

A disadvantage of the Ishibashi model is the fact that the material properties are only considered in the coefficients $C_1$ and $C_2$ (Table 1). These coefficients are constant for concrete, however, the material strength which largely affects the abrasion depth is not accounted for in the estimation of the abrasion volume.

4.2 Sensitivity of abrasion resistance coefficient $k_v$

Auel et al. (2015) discussed the sensitivity of $k_v$, stating that the coefficient is probably not a constant but dependent on the material properties based on statements from Chatanantavet and Parker (2009) and Turowski et al. (2013). Furthermore, Sklar and Dietrich (2004, 2012) showed that $k_v$ ranges from 1 to $9 \times 10^6$ for different materials. Fig. 8 shows $k_v$ as a function of $f_t$ for a number of research studies including rock and concrete samples. The splitting tensile strength for concrete is calculated using a correlation to the cubed compressive strength $f_c$ after Arioglu et al. (2006) and EN 1992-1-1 as:

$$f_t = 0.387 (0.8f_c)^{0.63} = 0.336f_c^{0.63} \quad [\text{MPa}] \quad [19]$$

Six rock and three concrete samples from Sklar and Dietrich (2004) are plotted as well as two theoretical bedrock values by Chatanantavet and Parker (2009). By means of a scaling analysis, the latter proposed $k_v = 10^4$ for weak and $k_v = 10^6$ for hard rock, respectively. Herein, $f_t = 1 \text{ MPa}$ and $15.5 \text{ MPa}$ is assumed for weak and hard rock, respectively, based on values given in Sklar and Dietrich (2001). The values from Auel (2014) are obtained from laboratory experiments using weak mortar as a substitute to allow for invert abrasion in short time. Values range from $k_v = 8 \pm 5 \times 10^2$ to $6 \pm 6 \times 10^3$. Data of Helbig et al. (2012) were obtained from rotating drum experiments. $30 \times 30 \text{ cm}$ concrete sample plates were mounted in a drum filled with both 10 kg of steel spheres and 10 kg of water. The drum rotates around a horizontal axis with a radius of 43 cm where the concrete samples immerge into the watersphere mixture at every rotation. Abrasion is measured after different time steps by both laser scanning and
weighing. All hydraulic parameters required to estimate $k_v$ from Eq. (12) can be calculated using the rotational velocity and the time the sample is submerged. However, one parameter, the saltation length $L_p$, turns out to be difficult to estimate. When the sample immerses into the water, the spheres start to impact the sample surface. Depending on the sphere size ($D = 4.4$ to 8 mm) about 4,782 to 28,744 spheres are present in the water corresponding to 0.3 to 2.1 spheres/cm$^3$. Additionally, due to the rotation, some spheres are transported towards a higher location inside the drum and drop back into the water. Both the large amount and the falling spheres lead to a very high impact rate. Hence it is assumed that $L_p$ is in the range of a millimeter. In Fig. 8, three data groups of Helbig et al. (2012) are plotted corresponding to $L_p = 0.1$, 1 and 5 mm, resulting in corresponding $k_v$ values of $3.1(±0.8)×10^6$, $3.1(±0.8)×10^5$ and $6.2(±1.7)×10^4$, respectively. A precise estimate of $L_p$ is difficult, but due to the large amount of particles in the drum, the values of 0.1 and 1 mm seem to be most adequate. Finally, the prototype data from Asahi SBT are calculated and additionally presented in Fig. 7. The data fit into the range of Helbig’s data with $L_p = 1$ mm.

![Fig. 7 Abrasion coefficient $k_v$ as a function of splitting tensile strength $f_t$.](image)

The data do not follow a clear trend. The sets of Auel (2014), Chatanantavet and Parker (2009), the rock set of Sklar and Dietrich (2004), and the prototype data of Asahi indicate an increase of $k_v$ with increasing tensile strength. However, the concrete data of Sklar and Dietrich (2004) show high $k_v$ values even for low $f_t$, and the data set of Helbig et al. (2012) largely depends on the estimation of $L_p$ and does not allow for precise analysis. Further investigation is needed to adequately define the correlation of $k_v$ to the invert material properties.

### 4.3 Auel model

Applying the prediction model given in Eq. (12) leads to the results given in Fig. 8. As the tunnel invert concrete strength varied over time (Fig. 3), Eq. (12) was applied for both material strengths, i.e. $f_t = 30$ MPa and 70 MPa, and averaged using the ratio given in Fig. 3. Auel et al. (2015) proposed an abrasion coefficient $k_v = 10^6$ if no additional abrasion measurement data are available. Applying this value leads to an underestimation of the real abrasion volume of about $δ = 80±7\%$ in average over 14 years (Fig. 8).

![Fig. 8 Measured abraded invert volume of Asahi SBT from 1998 to 2011 and comparison with abrasion volume predictions by Auel et al. (2015) based on $k_v$ values calibrated from the 1998 to 2006 data.](image)

However, a calibration is possible, as the real abrasion at Asahi SBT is known. The total measurement period from 1998 to 2011 has been divided into a calibration period (1998 to 2006) and a validation period (2007 to 2011). The abrasion coefficient has been calculated for the calibration period, leading to a value of $k_v = 2.0×10^5$. Applying this value to the validation period leads to the results given in Fig. 8. The abrasion volumes in 2008 and 2010 are of minor extent leading to an overestimation of 54% and 26%, respectively. However, neglecting these two values, thus taking into account only the three years with considerable abrasion in 2007, 2009, and 2011 leads to a deviation of only $δ = 12±4\%$. 

![Fig. 9 Calibration of the abrasion model using the prototype data of Asahi SBT.](image)
5 Conclusion

Herein, the two abrasion prediction models by Ishibashi (1983) and Auel et al. (2015) developed for hydraulic structures prone to supercritical flows are compared. Both models are validated using prototype data from the Asahi sediment bypass tunnel (SBT) in Japan showing good agreement if adjusted and calibrated. The prediction model by Ishibashi (1983) reveals certain inconsistencies in its grinding stress term accounting for the cutting wear. It is proposed to omit this term in case of brittle materials such as concrete. The model leads to good results if the grinding stress is not considered.

The prediction model by Auel et al. (2015) shows excellent performance if adjusted to the prototype measurements. However, if prototype abrasion measurements are not available, a $k_v$ value has to be assumed. The analysis presented herein leads to the conclusion that $k_v$ is lower for concrete than the widely applied value of $10^6$ valid for bedrock abrasion. A value around $2.0 \times 10^6$ seems to be more adequate in case of Asahi SBT. However, more research on prototype hydraulic facilities as well as laboratory experiments should be conducted to amplify the knowledge of the variation of $k_v$. Hagmann et al. (2015) conduct research at three Swiss SBTs where abrasion volumes as well as sediment transport are measured. These results are expected to complement and enhance the present results.

Independent of the prediction model, utmost focus has to be put on the estimation of the sediment bed-load transport as both models are directly correlated to it. The theoretical transport capacity is not always adequate, as the real transport may be much lower. The transport has to be calibrated using prototype data such as periodical sedimentation survey data of the reservoir or direct measurements such as geophones or plate microphones (Hagmann et al. 2015, Koshiba 2015). Furthermore the upstream riverbed slope, the river cross-sections as well as the particle size distribution should be analyzed as precisely as possible. In the case of the Asahi River, the theoretical capacity applying local values of bed slope and river width surveyed directly upstream of the SBT intake (Awazu 2015) lead to about 9 times higher theoretical sediment transport capacity as the data evaluated by Newjee Inc. using the real reservoir survey data as calibration. This large difference underlines the need for precise estimation of the real sediment transport impacting the hydraulic structure.

Acknowledgments

The support of Sohei Kobayashi and Takahiro Koshiba regarding the translation of Ishibashi’s work is gratefully acknowledged. Furthermore, the first author acknowledges the financial support of the Japanese Society for the Promotion of Science (JSPS).

**Notation**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_p$</td>
<td>vertical abrasion rate</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$B$</td>
<td>tunnel width</td>
<td>[m]</td>
</tr>
<tr>
<td>$C_1, C_2$</td>
<td>material property constants</td>
<td>[m²/(kgf)]</td>
</tr>
<tr>
<td>$D$</td>
<td>particle diameter</td>
<td>[m]</td>
</tr>
<tr>
<td>$D_s$</td>
<td>dimensionless grain size</td>
<td>[-]</td>
</tr>
<tr>
<td>$E_i$</td>
<td>kinetic energy of a single particle</td>
<td>[kgf]</td>
</tr>
<tr>
<td>$E_k$</td>
<td>kinetic energy by saltating particle</td>
<td>[kgf m]</td>
</tr>
<tr>
<td>$F_i$</td>
<td>impact force</td>
<td>[kgf]</td>
</tr>
<tr>
<td>$f_c$</td>
<td>cubed compression strength</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$f_s$</td>
<td>splitting tensile strength</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration</td>
<td>[m/s²]</td>
</tr>
<tr>
<td>$h$</td>
<td>flow depth</td>
<td>[m]</td>
</tr>
<tr>
<td>$H_p$</td>
<td>particle saltation height</td>
<td>[m]</td>
</tr>
<tr>
<td>$I$</td>
<td>number of particle impacts</td>
<td>[1/m]</td>
</tr>
<tr>
<td>$k$</td>
<td>bed roughness height</td>
<td>[m]</td>
</tr>
<tr>
<td>$k_1, k_2$</td>
<td>auxiliary parameter</td>
<td>[m²/kgf]</td>
</tr>
<tr>
<td>$k_r$</td>
<td>rock resistance coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$L$</td>
<td>total invert length</td>
<td>[m]</td>
</tr>
<tr>
<td>$L_p$</td>
<td>particle saltation length</td>
<td>[m]</td>
</tr>
<tr>
<td>$M_p$</td>
<td>particle mass</td>
<td>[kg]</td>
</tr>
<tr>
<td>$M_p^*$</td>
<td>submerged particle weight</td>
<td>[kg]</td>
</tr>
<tr>
<td>$N_i$</td>
<td>$L/L_p = \text{impact frequency}$</td>
<td>[-]</td>
</tr>
<tr>
<td>$n_i$</td>
<td>amount of particles per volume</td>
<td>[1/m³]</td>
</tr>
<tr>
<td>$n_1$</td>
<td>auxiliary parameter</td>
<td>[kgf/m²]</td>
</tr>
<tr>
<td>$P_R$</td>
<td>rolling probability</td>
<td>[-]</td>
</tr>
<tr>
<td>$Q_d$</td>
<td>design discharge</td>
<td>[m³/s]</td>
</tr>
</tbody>
</table>
$Q_s$ gravimetric bedload rate [kg/s]
$q_s$ specific gravimetric bedload rate [kg/(sm)]
$s$ density ratio $s = \frac{\rho_s}{\rho}$ [-]
$S_b$ bed slope [-]
$U$ uniform flow velocity [m/s]
$U_p$ horizontal particle velocity [m/s]
$U_*$ friction velocity $U_* = (gR_sS)^{0.5}$ [m/s]
$V_a$ abraded volume [m$^3$]
$V_t$ amount of transported sediment [m$^3$]
$V_s$ particle settling velocity [m/s]
$W_{im}$ vertical particle impact velocity [m/s]
$W_f$ friction work due to grinding stress [kgf m]
$Y_M$ Young’s Modulus of elasticity [Pa]
$\beta$ auxiliary parameter [m/(kgf)$^{2/3}$]
$\gamma_{nm}$ particle impact angle [-]
$\delta$ relative change [%]
$\theta$ Shields parameter $\theta = U_*^2 [(s-1)gD]$ [-]
$\theta_c$ critical Shields parameter [-]
$\lambda_p$ air porosity [-]
$\mu_s$ dynamic friction coefficient [-]
$\nu$ kinematic viscosity [m$^2$/s]
$\rho$ fluid density [kg/m$^3$]
$\rho_s$ particle density [kg/m$^3$]

References


*(Received June 11, 2015)*